The holographic stress tensor

jueves, 24 de noviembre de 2022 12:58

This analysis can also be done for bulk gauge fields, and for the gravitational field.

$$ds^2 = \frac{dz^2 + \eta_{\mu\nu}\, dx^\mu dx^\nu}{z^2} \qquad \mu,\nu = 0,\dots,d-1$$

Boundary metric: $\dfrac{\eta_{\mu\nu}}{z^2} \to \dfrac{\eta_{\mu\nu} + h_{\mu\nu}(x,z)}{z^2}$

That is $\delta g_{zz} = 0 \qquad \delta g_{\mu\nu} = \dfrac{h_{\mu\nu}(x,z^2)}{z^2}$

$\delta g_{z\mu} = 0$

Fefferman-Graham gauge (Gaussian normal)

Solve gravitational (perturbation) eqns:

$$h_{\mu\nu}(x,z) = \underbrace{h_{\mu\nu}(x,0)}_{\text{source}} + z^d \underbrace{\langle T_{\mu\nu}\rangle(x)}_{\text{response}} + \dots$$

more generally:

$$\eta_{\mu\nu} \to g_{\mu\nu}(x,z) = g^{(0)}_{\mu\nu}(x) + z^2 g^{(1)}_{\mu\nu}(x) + z^4 g^{(2)}_{\mu\nu}(x) + \dots$$

FG expansion:

$$\langle T_{\mu\nu}\rangle = g^{(d/2)}_{\mu\nu}(x)$$

$g^{(i)}_{\mu\nu} \qquad 0 < i < \frac{d}{2}$ are determined in terms of $g^{(0)}_{\mu\nu}$ and its curvature

$g_{\mu\nu}$ $\quad 0 < i < \frac{d}{2}$ are determined in terms of $g^{(0)}_{\mu\nu}$ and its curvature

In the CFT dual:

$$I = I_{CFT} + \frac{1}{2}\int d^dx \, h_{\mu\nu}(x)\, T^{\mu\nu}_{CFT}(x)$$

Stress Tensor of field Theory measures its response to metric deformations

z^d : massless field

This is not different than the way we're used to measure mass (energy) in gravity.

Take $z = 1/r$ $\quad$ (or $z = L^2/r$)

Then

$$ds^2 = \frac{dr^2}{r^2} + \left(r^2 \eta_{\mu\nu} + \frac{\langle T_{\mu\nu}\rangle}{r^{d-2}} + \dots\right) dx^\mu dx^\nu$$

In asymptotically flat space, we expand near infinity where the field is weak ~ Newtonian

$$ds^2 = dr^2 - \left(1 + \frac{c_d M}{r^d} + \dots\right) dt^2 + r^2 d\Omega + \dots$$

$$\downarrow$$

$$\frac{\langle T_{tt}\rangle}{r^{d-2}} \qquad c_d =$$

In AdS (global)

$$ds^2 = \frac{dr^2}{r^2} - \left(r^2 + 1 - \frac{c_d M}{r^d} + \dots\right) dt^2 + r^2 d\Omega + \dots$$

So the formula above for $\langle T_{\mu\nu}\rangle_{CFT}$ simply identifies the stress-energy from the leading decay of the gravitational field.

We have seen that $\langle T_{\mu\nu}\rangle$ can be obtained from the metric asymptotic expansion in a particular gauge. But we also saw that it's possible to obtain it in a covariant, gauge-independent manner, from the extrinsic curvature of the boundary metric.

A similar analysis yields the Brown-York stress tensor as

$$T_{\mu\nu} = \frac{1}{8\pi G}\left(K_{\mu\nu} - \gamma_{\mu\nu} K\right) \qquad \gamma_{\mu\nu} = g^{(0)}_{\mu\nu}(x)$$

This is divergent. We can remove the divergence by subtracting the empty AdS term for the same boundary metric.

But we'll see that there's another option, motivated by the duality to a CFT