Geometry of AdS

jueves, 24 de noviembre de 2022 12:57

Let's begin very simple:

Sphere:

Surface $x_1^2+x_2^2+x_3^2=1$ in $ds^2=dx_1^2+dx_2^2+dx_3^2$

is a sphere S^2 : constant positive unit curvature

Symmetry $SO(3)$ is manifest

Hyperboloid: $-x_1^2+x_2^2+x_3^2=-1$ in $ds^2=-dx_1^2+dx_2^2+dx_3^2$

$$SO(2,1)$$

Now with Time

deSitter$_{1+1}$ $-x_1^2+x_2^2+x_3^2=+1$ in $ds^2=-dx_1^2+dx_2^2+dx_3^2$

$$SO(1,2) \sim SL(2,\mathbb{R})$$

Anti-deSitter$_{1+1}$

$$-x_1^2-x_2^2+x_3^2=-1 \quad \text{in} \quad ds^2=-dx_1^2-dx_2^2+dx_3^2$$

$$SO(2,1) \sim SL(2,\mathbb{R})$$

Sphere S^n with $\bar{x}_{n+1}=(x_1,\dots,x_{n+1})$

$$\bar{x}_{n+1}^2=1 \quad \text{in} \quad ds^2=d\bar{x}_{n+1}^2$$

$$SO(n)$$

Anti-deSitter$_n$

$$-X_1^2 - X_2^2 + \bar{X}_{n-1} = -1 \quad \text{in} \quad ds^2 = -dX_1^2 - dX_2^2 + d\bar{X}_{n-1}^2$$

$$SO(2, n-1)$$

Coordinates:

$$\underbrace{-X_1^2 - X_2^2}_{\text{polars}} + \underbrace{\bar{X}_{n-1}}_{\text{spherical}} = -1 \quad \text{in} \quad ds^2 = -dX_1^2 - dX_2^2 + d\bar{X}_{n-1}^2$$

$$x_1 = \sqrt{r^2+1}\,\cos t \qquad \bar{X}_{n-1}^2 = r^2$$
$$x_2 = \sqrt{r^2+1}\,\sin t$$

$$\Rightarrow \quad ds^2 = -(r^2+1)\,dt^2 + \frac{dr^2}{r^2+1} + r^2\, d\Omega_{n-2}$$

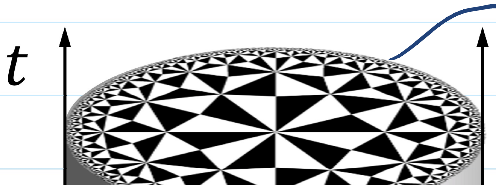
unwrap t: global covering of AdS

With curvature radius L:

$$ds^2 = -\left(\frac{r^2}{L^2}+1\right) dt^2 + \frac{dr^2}{\frac{r^2}{L^2}+1} + r^2\, d\Omega_{n-2}$$

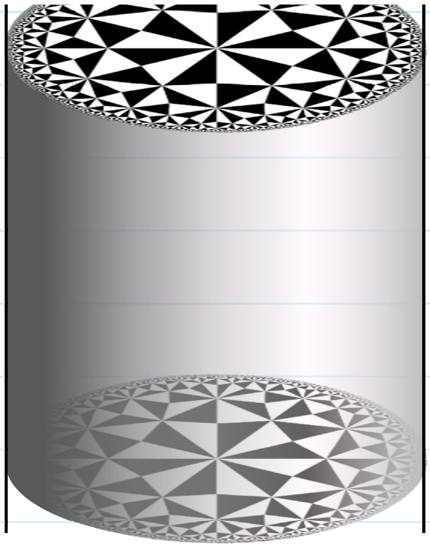
Spatial part:

$$ds^2 = -(r^2+1)\,dt^2 + \underbrace{\frac{dr^2}{r^2+1} + r^2\, d\Omega_{n-1}}_{\text{Hyperboloid } \mathbb{H}_{n-1}}$$



$\to \mathbb{H}_{n-1}$

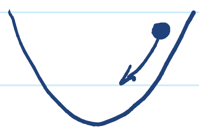
conformal to



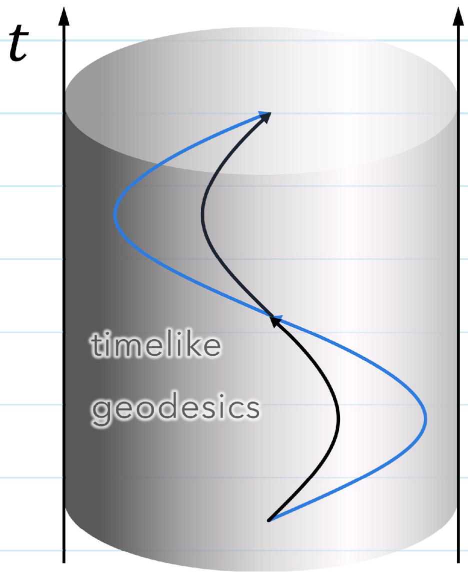
conformal to
Poincaré disk/sphere

Time:

$$ds^2 = \underbrace{-(r^2+1)\,dt^2}_{\text{gravitational potential}} + \frac{dr^2}{r^2+1} + r^2 d\Omega_{n-1}$$



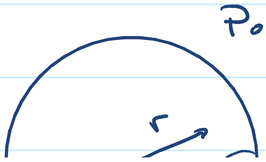
"isochrone pendulum"

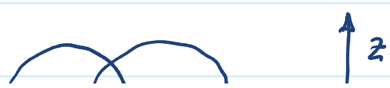


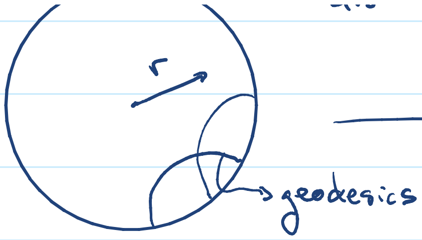
Poincaré upper-half plane representation

Poincaré disk

Poincaré UHP



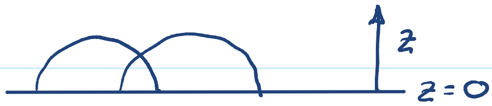




$$ds^2 = -(r^2+1)\,dt^2 + \frac{dr^2}{r^2+1} + r^2 d\Omega_{n-2}$$

boundary $\mathbb{R}_t \times S^{n-2}$

(t, Ω_{n-2})



$$ds^2 = \frac{dz^2 - dt^2 + d\bar{x}^2_{n-2}}{z^2}$$

(find the embedding)

boundary $\mathbb{R}_t \times \mathbb{R}^{n-2}$

$x^\mu = (t, \bar{x}_{n-2})$

$\downarrow$

Minkowski$_{n-1}$

Not a global cover: horizon at $z = \infty$